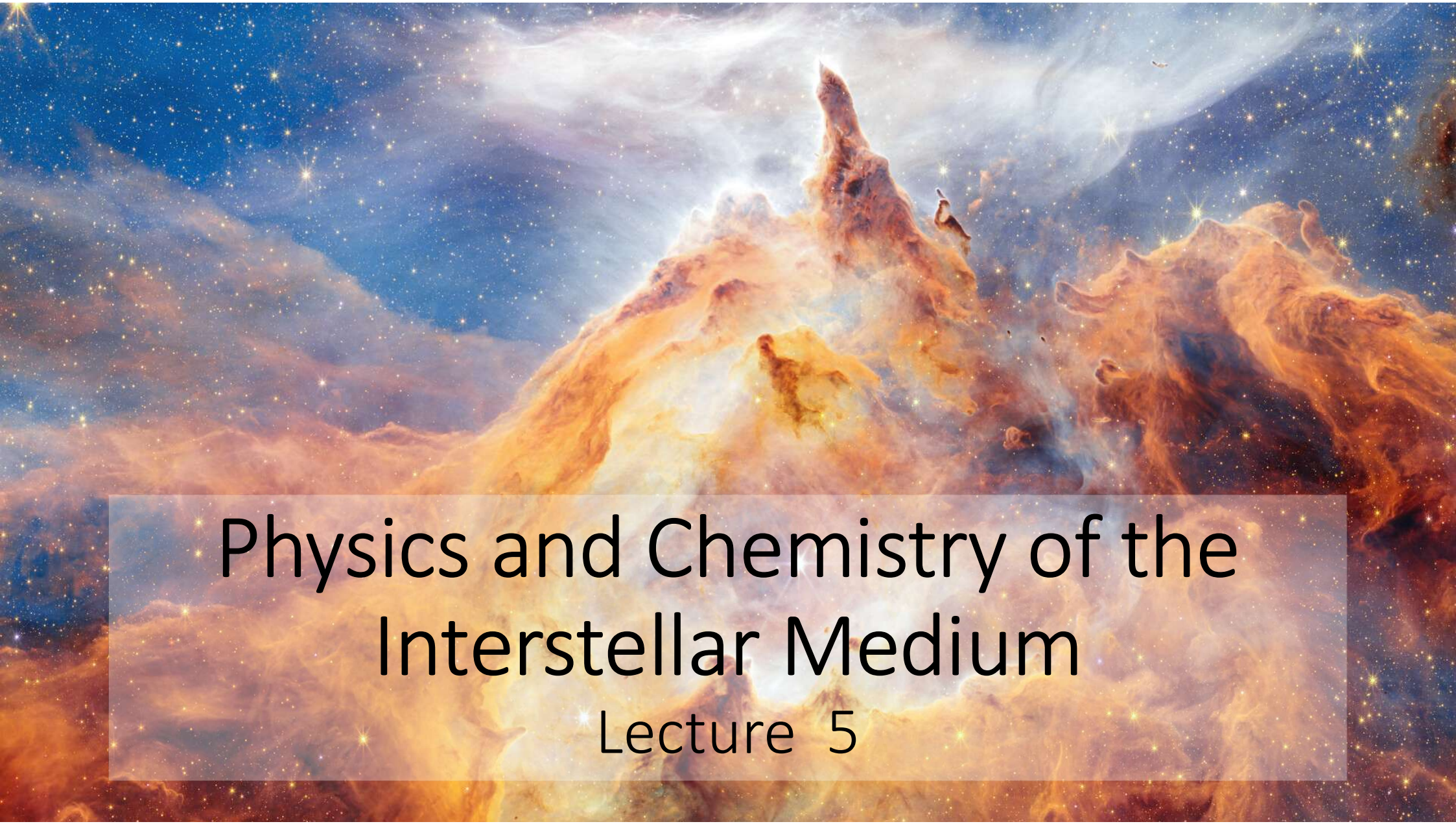


Physics and Chemistry of the Interstellar Medium

Lecture 5

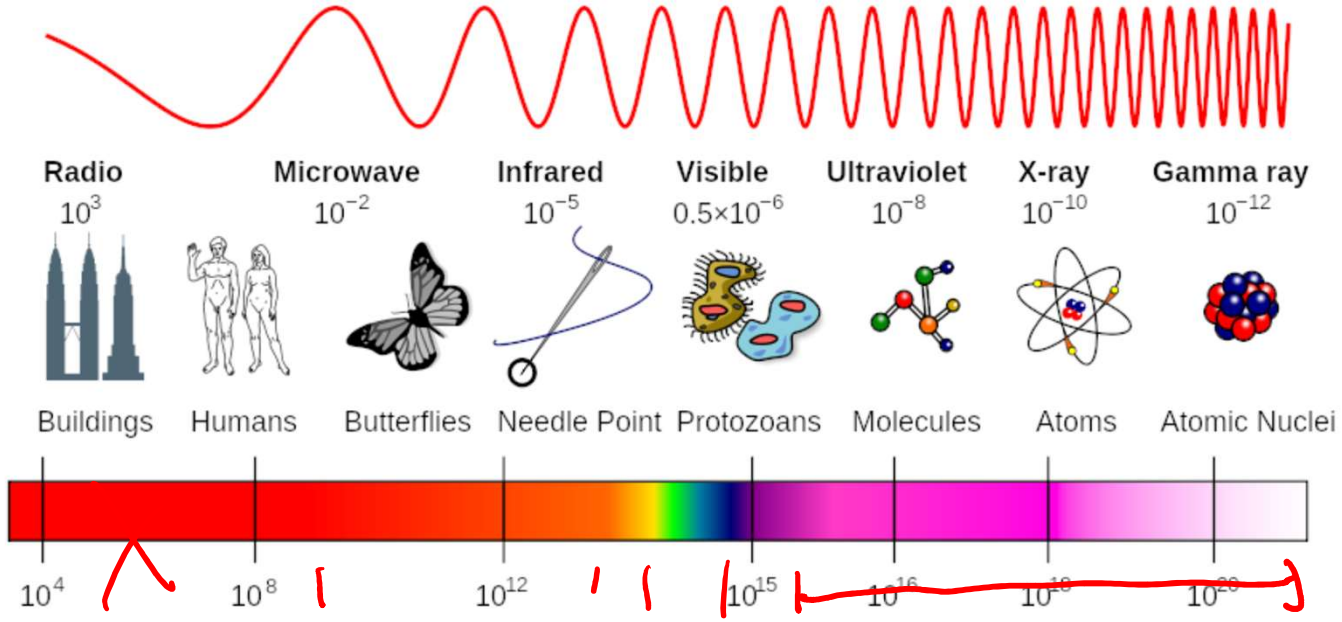
Radiation

Lecture V

3.1. Dirac perturbation theory

3.2. Discrete systems

- The hydrogen atom
- Fine structure
- Hyperfine structure
- Multiple electrons
- Molecules



Radiation

Motivation

- **Fundamentals**
  - Interaction of the electromagnetic free-field with matter
- **Diagnostics**
  - Radiation is our only way to obtain information from astrophysical systems
  - Imprint of the properties of the interstellar matter on the photon field
  - All analysis in this lecture is based on inverting the radiative transfer problem
- **Energy Balance**
  - Radiative contributions Λ, Γ dominate the energy balance in many regions
  - Emission and absorption needs to be understood

Basic Physics

- **Dirac Perturbation Theory**
  - Atomic system with Hamiltonian $\hat{H}^0_{at}$ and eigenstates $|a_{at}\rangle$
    - Eigenvalues : $E_{a,at}$
  - Free photon field with eigenstates $|n_\lambda\rangle$ corresponding to n_λ photons of wavelength λ
    - Eigenvalues

$$E_{Free-field} = \sum_\lambda \frac{hc}{\lambda} n_\lambda = \sum_\lambda h\omega n_\lambda \qquad \omega = 2\pi\frac{c}{\lambda}$$

  - Total Hamiltonian

$$\hat{H}^0 = \hat{H}^0_{at} + \hat{H}^0_{Free-field}$$

    - with eigenstates

$$|a\rangle = |a_{at}\rangle|n_\lambda\rangle$$

  - **Interaction** $\hat{H}_i$
    - As perturbation term $\hat{H}_i \ll \hat{H}^0$

Dirac Perturbation theory

- **Interaction**
  - Transition from state $|a\rangle$ to state $|a'\rangle$ determined by Fermi's Golden Rule:

$$W_{a,a'} = \frac{2\pi}{\hbar} |\langle a'|\hat{H}_i|a\rangle|^2 \delta(E_{a'} - E_a)$$

$W_{a,a'}$: Transition probability
δ : function from energy conservation

$E_a, E_{a'}$: Eigenvalues of unperturbed system given by Hamiltonian $\hat{H}^0$

- **Computation of interaction matrix** $\langle a'|\hat{H}_i|a\rangle$
  - Through Maxwell equations
  - Without prove here (see QM II)

Interaction matrix

$$\langle a'|\hat{H}_i|a\rangle = \sum_\lambda \frac{e}{m_{at}}\sqrt{\frac{\hbar}{2\omega\epsilon_0}}\vec{E}_{phot}\cdot \left\{\sqrt{n_\lambda+1}\langle a'_{at}|\vec{p}e^{i\vec{k}\vec{r}}|a_{at}\rangle\langle n'_\lambda|n_\lambda+1\rangle + \sqrt{n_\lambda}\langle a'_{at}|\vec{p}e^{-i\vec{k}\vec{r}}|a_{at}\rangle\langle n'_\lambda+1|n_\lambda\rangle\right.$$

with $\omega = 2\pi\frac{c}{\lambda} = c|\vec{k}|$

Change of the free field characterized by the change of the photon number:

$\langle n'_\lambda|n_\lambda+1\rangle = \delta(n'_\lambda - [n_\lambda+1])$: n_λ must increase by 1 → emission

$\langle n'_\lambda+1|n_\lambda\rangle = \delta([n'_\lambda+1] - n_\lambda)$: n_λ must decrease by 1 → absorption

Fermi condition $\delta(E_a - E_{a'})$ requires that energy of atomic system changes in opposite way:

$$E_{a,at} = E_{a',at} \pm \hbar\omega$$ → **Energy conservation**

Interaction matrix

- Transition strength
  - Strength of the coupling given by electromagnetic interaction term $\frac{e}{m_{at}}\vec{p}\cdot\vec{E}_{photo}$
  - $\frac{e}{m_{at}}\vec{p} = e\dot{\vec{r}}$ = charge momentum of the atomic system coupling to the photon electric field
  - Apart from the energy conservation rule, only the properties of the atomic system determine the transition probabilities

→ Selection rules

- Consider series expansion: $e^{\pm i\vec{k}\vec{r}} = 1 \pm i\vec{k}\vec{r} - \frac{\vec{k}^2\vec{r}^2}{2} + \dots$
- Interaction terms: $\left\langle a'_{at}\left|\frac{e}{m_{at}}\vec{p}e^{\pm i\vec{k}\vec{r}}\right|a_{at}\right\rangle = \langle a'_{at}|e\dot{\vec{r}}|a_{at}\rangle \pm \langle a'_{at}|ei\vec{k}(\vec{r}\times\dot{\vec{r}})|a_{at}\rangle - \dots$

Aufspaltung in symmetrischen und antisymmetrischen Anteil

Radiation

Selection rules

- **First order term** $\left\langle a'_{at} \left| e\dot{\vec{r}} \right| a_{at} \right\rangle$
  - Interpretation:
    - $e\dot{\vec{r}}$ must change between $|a_{at}\rangle, |a'_{at}\rangle$ to provide a non-zero expectation value
    - $e\ddot{\vec{r}} \neq 0$
    - **Charge accelerated/decelerated** → trivial knowledge confirmed
    - Change must happen resonantly for states $|a_{at}\rangle, |a'_{at}\rangle$ → frequency ω_λ
    - Precondition: Dipole moment $\vec{d} = e\vec{r}$ must exist
  - Dipole transition:

$$\left\langle a'_{at} \left| e\dot{\vec{r}} \right| a_{at} \right\rangle \sim \vec{d} \cdot \omega$$

- **Next terms:**
  - $e\left(\vec{r} \times \dot{\vec{r}}\right) = \vec{M}$ = magnetic moment → magnetic dipole transition
  - $\vec{Q}$ = electric quadrupole …

Transition probability

- **Combine everything for dipole transition and emission case:**

$$W_{a,a'} = \sum_{\lambda} \frac{2\omega^2}{3\hbar c^3 \epsilon_0} \left|\langle d_{a,a'} \rangle\right|^2 (1 + n_\lambda)$$

- **Express in terms of Einstein coefficients:**
  - Spontaneous emission: $A_{a,a'} = W_{a,a'}(n_\lambda = 0) \times \frac{\omega}{2\pi}$
  - Induced emission: $B_{a,a'} = \frac{W_{a,a'}(n_\lambda - 1)}{u_\omega} \times \frac{\omega}{2\pi}$
    - with radiative energy density

$$u_\omega = \int_{4\pi} I_\omega d\Omega$$

I_ω: intensity

Einstein coefficients

- $W_{a,a'} = \sum_\lambda \frac{2\omega^2}{3\hbar c^3 \epsilon_0} \left|\langle d_{a,a'} \rangle\right|^2 (1 + n_\lambda)$
  - Spontaneous emission: $A_{a,a'} = W_{a,a'}(n_\lambda = 0) \times \frac{\omega}{2\pi} = \frac{\omega^3}{3\pi\hbar c^3 \epsilon_0} \left|d_{a,a'}\right|^2$
  - Induced emission: $B_{a,a'} = \frac{W_{a,a'}(n_\lambda - 1)}{u_\omega} \times \frac{\omega}{2\pi}$
    - using the photon density: $n_\lambda = \frac{\pi^2 c^3}{\hbar\omega^3} u_\omega$
  - **Result**: $B_{a,a'} = \frac{\pi^2 c^3}{\hbar\omega^3} A_{a,a'} = \frac{\pi}{3\hbar^2 \epsilon_0} \left|d_{a,a'}\right|^2$

Einstein coefficients

- **Absorption case:**

$$W_{a,a'} = \sum_{\lambda} \frac{2\omega^2}{3\hbar c^3 \epsilon_0} \left|\langle d_{a,a'} \rangle\right|^2 (n_\lambda)$$

  - Analogously:

$$B_{a',a} = \frac{\pi}{3\hbar^2 \epsilon_0} |d_{a'a}|^2 = B_{a,a'} \frac{|d_{a',a}|^2}{|d_{a,a'}|^2}$$

  - With dipole moment of inverse transition

- **Higher moments:**
  - Magnetic dipole $A_{a,a'} = \frac{\omega^3}{3\pi\hbar c^3 \mu_0} |\vec{M}|^2$
  - Electric quadrupole $A_{a,a'} = \frac{\omega}{40\pi\hbar c^5 \epsilon_0} |\vec{\vec{Q}}|^2$

Einstein coefficients

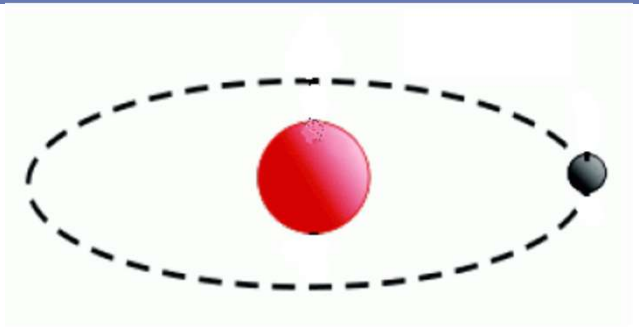
- No assumptions were made about the atomic system!
  - Only requirements:
    - Energy conservation $\delta(E_a - E_{a'})$
    - Selection rules=Harmonic variation of $|d_{a,a'}|, |M_{a,a'}|, |Q_{a,a'}|$
  - **Einstein notation valid for any interaction of radiation with matter!**
    - Representation of the solution of the Dirac equation
  - Can be used for all radiative processes in the ISM
    - Atoms
    - Molecules
    - Plasma
    - Dust
- **Start: discrete systems**

Discrete systems

Basic atomic physics:

The H-atom

- Simple problem: p + e$^-$
  - Pure Coulomb problem: $\hat{H} = \frac{\vec{p}}{2m_r} - \frac{(Z)e^2}{4\pi\epsilon_0|\vec{r}|}$
  - With reduced mass $m_r = \frac{m_e m_p}{m_e + m_p}$
  - Solution has eigenvalues $E = -R\frac{Z^2}{n^2}$
  - With Rydberg constant $R = \frac{1}{2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{m_r}{\hbar^2} \times 13.605\,eV$



$\frac{m_e}{m_p} = \frac{1}{1836}$

Z = charge for more general case of multiple protons

n = main quantum number

Repetition of basic atomic physics

The H-atom

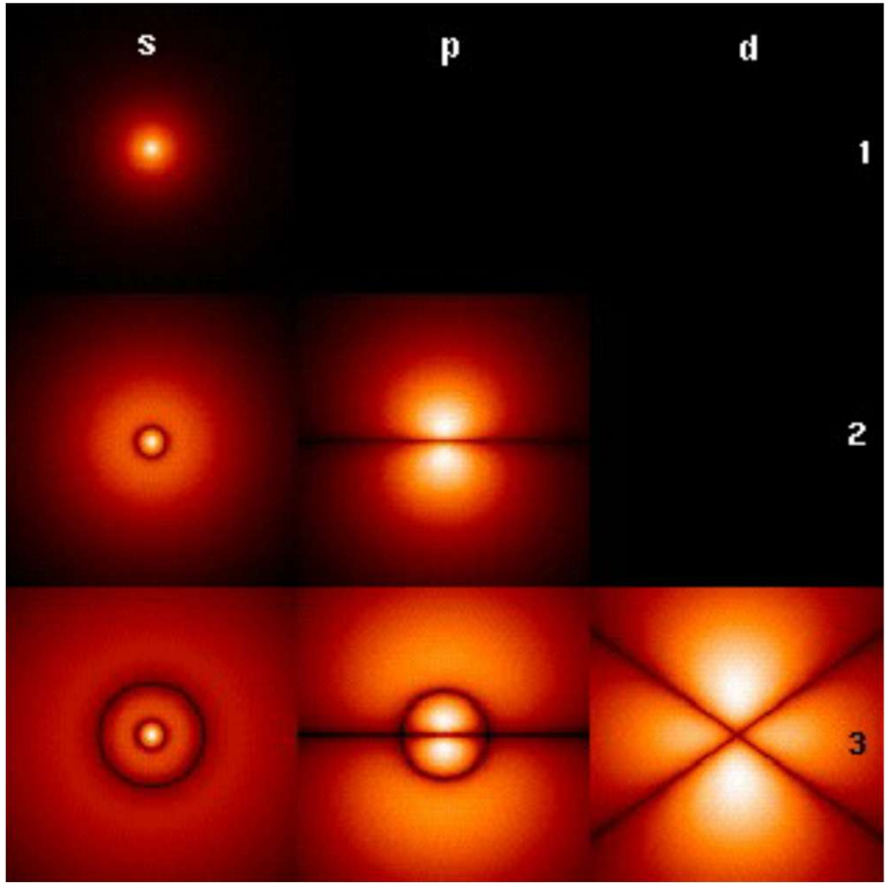
- **Orbits**: Solution of the full spatial problem
- Provides in total 3 quantum numbers: n, l, m
  - l = orbital quantum number $l = 0, \ldots, n-1$
  - m_l = projection of l on z-axis: $m_l = -l, \ldots, l$
  - Corresponding eigenvector: $|n, l, m_l\rangle$
  - **Energy levels n are degenerate with respect to l and m_l**
  - Level of degeneracy = statistical weight of energy level

$n = 1: l = 0, m_l = 0 \quad \rightarrow g = 1$

$n = 2: l = 0, m_l = 0$

$\quad\quad l = 1, m_l = -1, 0, 1 \quad \rightarrow g = 1 + 3 = 4$

$n = 3: l = 0,1,2, \ldots. \quad \rightarrow g = 9$

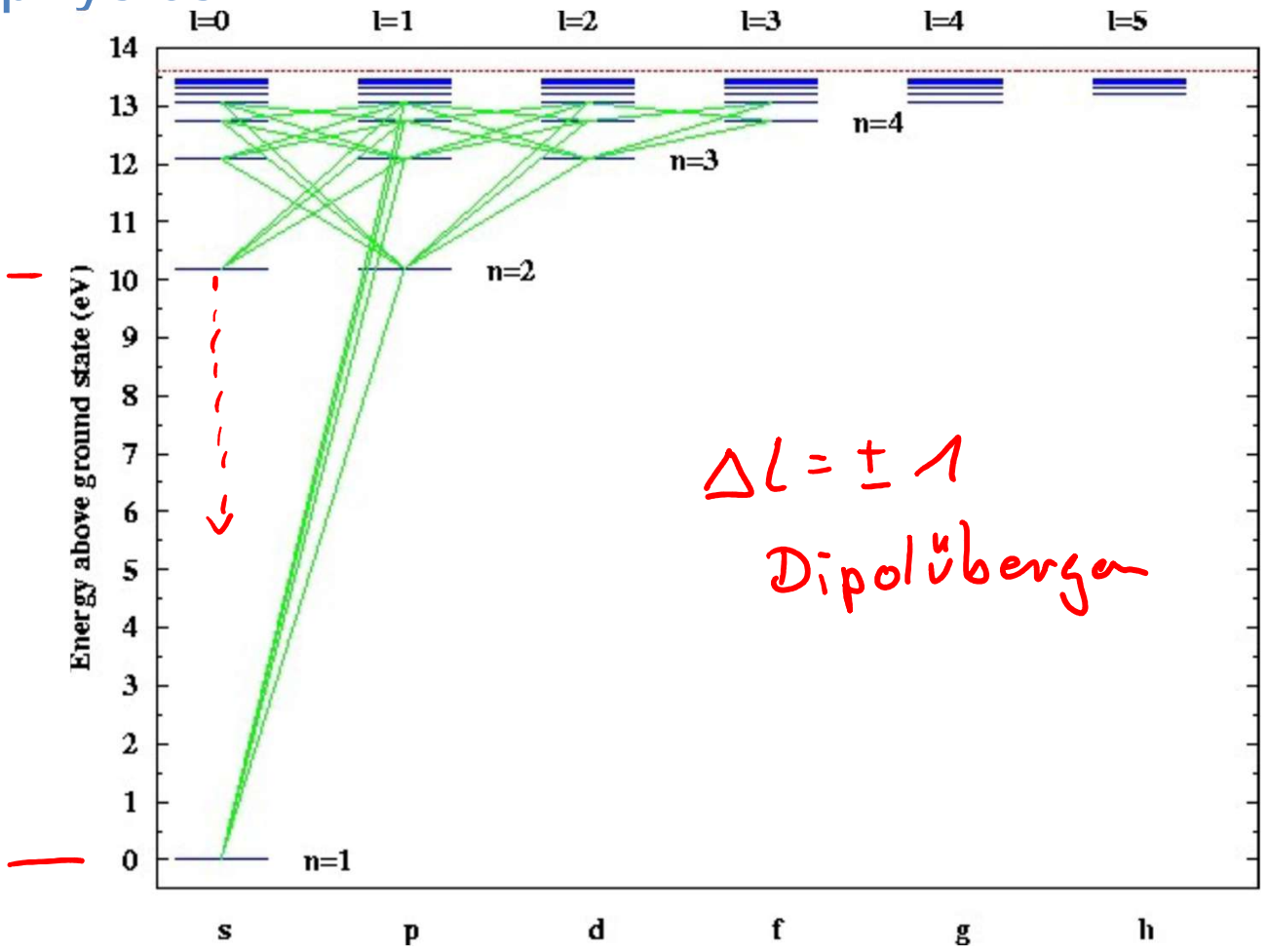


Repetition of basic atomic physics

- The H-atom

$$E_n - E_{n'} = (-Z^2) R \left(\frac{1}{n^2} - \frac{1}{n'^2} \right)$$

- Possible transitions
  - from level n to n'



Basics of spectroscopy

Repetition of basic atomic phy

- The H-atom (Z=1)
- Possible transitions
  - from level n to n'

$$\Delta E = -R\left(\frac{1}{n^2} - \frac{1}{n'^2}\right)$$

- Gives series for different n'
  - $n' = 1$ - Lyman
  - $n' = 2$ - Balmer
  - $n' = 3$ - Paschen
  - $n' = 4$ - Bracket
  - $n' = 5$ - Pfund

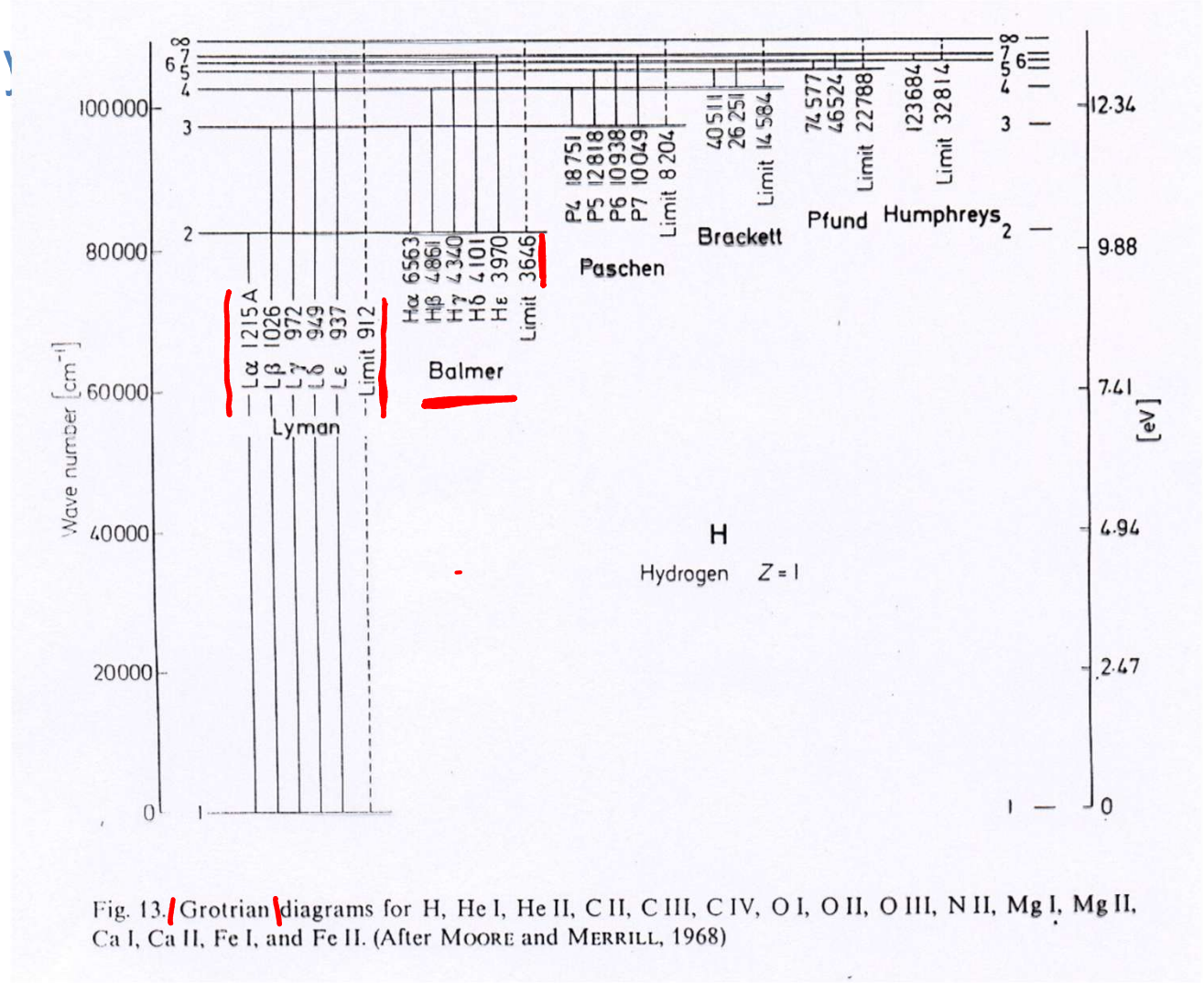


Fig. 13. Grotrian diagrams for H, He I, He II, C II, C III, C IV, O I, O II, O III, N II, Mg I, Mg II, Ca I, Ca II, Fe I, and Fe II. (After Moore and Merrill, 1968)

Fine structure

The electron spin

- Adds spin system $|s, m_s\rangle$ with $s = 1/2$
- Provides additional coupling term between orbital momentum and spin

$$\hat{H}_{FS} = \frac{e}{2m_e^2 c^2}\left(-\frac{1}{r}\frac{\partial}{\partial r}\frac{(Z)e^2}{4\pi\varepsilon_0 |\vec{r}|}\right)\hat{\vec{s}}\hat{\vec{L}}$$

- Solution has eigenvalues

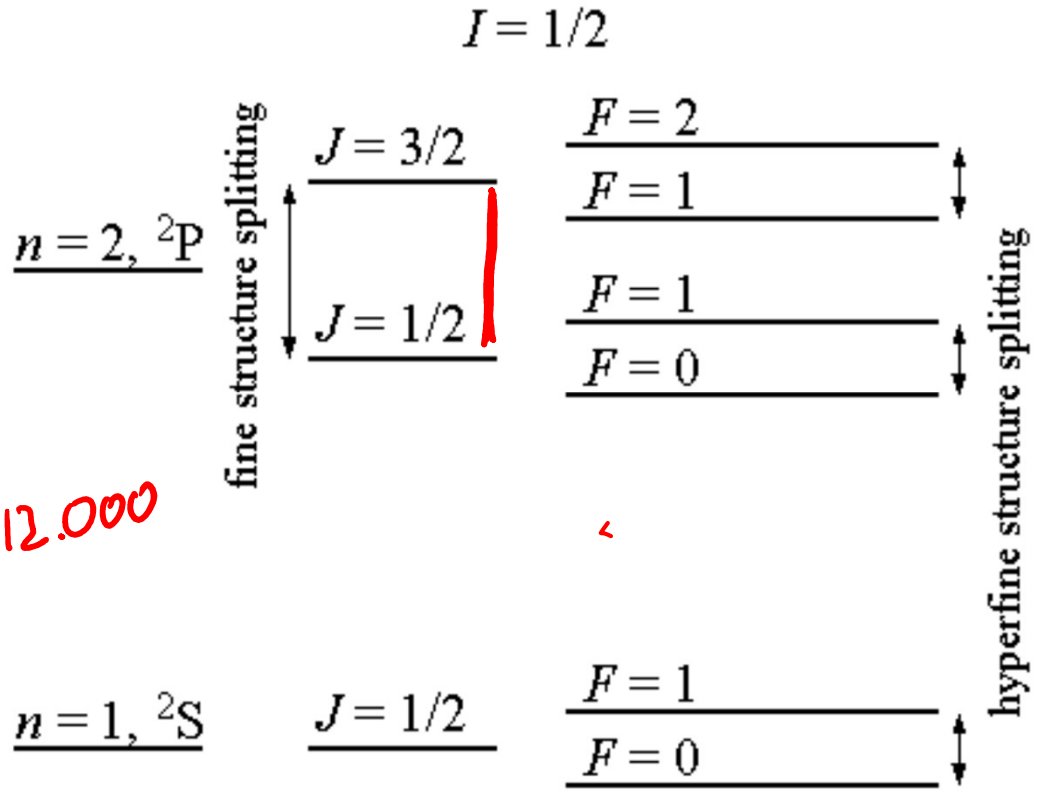
$$E_{FS} = \alpha^2 R \frac{Z^4}{n^3}\frac{j(j+1) - l(l+1) - s(s+1)}{l(l+1)(2l+1)}$$

- j = total angular momentum from $\vec{j} = \vec{l} + \vec{s} \;\rightarrow\; j = l - s, \ldots, l + s$
- $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}$ Sommerfeld fine structure constant

Fine structure

The electron spin

- Fine structure energies are scaled by α^2 relative to main level energies
- H-atom
  - No split for $n = 1, l = 0, s = 1/2$ because $j = 1/2$ independent of m_s
  - First split for $n = 2, l = 1, s = \frac{1}{2}$
    $\rightarrow j = 1/2\ , 3/2$
    $\Delta E_{fs} = 4.53 \times 10^{-5} eV$
    $= 10.9\ GHz$
    - Radio frequency range
    - Requires $n = 2$ excitation: 10.2 eV = 120 000 K
    - Not directly observable due to other splits



1eV ≈ 12.000

Basics of spectroscopy: Notation

Spectroscopy notation

$^{2s+1}l_j^{(p)}$

- l encrypted in letters
  - $0=s, 1=p, 2=d, 3=f, 4=g, 5=h, \ldots$
- p=parity
  - Blank for parity=even, o for parity = odd
  - Characterizes whether wave function changes sign under reflection of all electron positions through the origin.
- Examples:
  - $^2s_{1/2}$= ground state
  - $^2p_{1/2}$, $^2p_{3/2}$= first excited state (Balmer level) + $^2s_{1/2}$, $n=2$

Hyperfine splitting

The nuclear spin

- Adds nuclear spin system $|i, m_i\rangle$ with $i = 1/2$ for every nucleon
- Treatment equivalent to fine-structure splitting
- New operator: $\vec{f} = \vec{j} + \vec{i}$ and corresponding quantum number f
  - f = total angular momentum from → $j - i, \ldots, j + i$
- Eigenvalues of hyperfine coupling

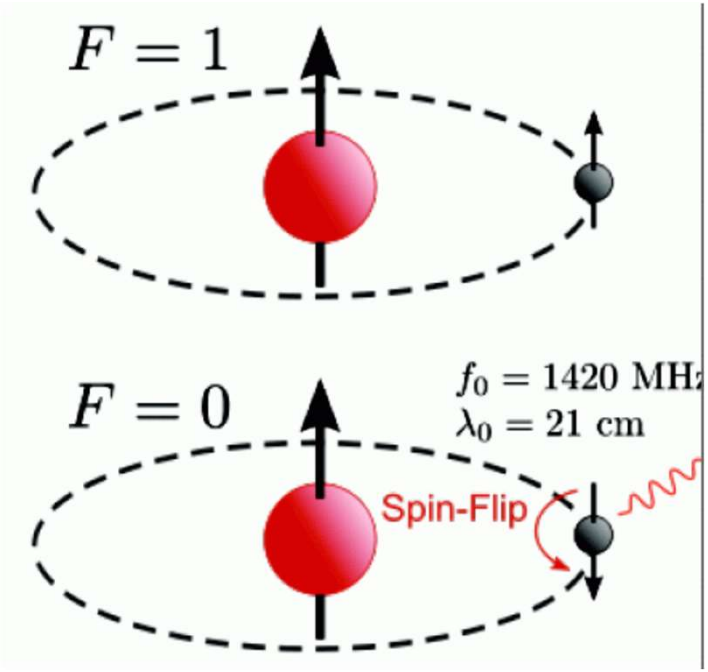
$$E_{HF} = g_N \left(\frac{m_e}{m_p}\right) \alpha^2 R \frac{Z^4}{n^3} \frac{f(f+1) - j(j+1) - i(i+1)}{j(j+1)(2j+1)}$$

  - Hyperfine splitting typically lower than fine-structure splitting by factor m_e/m_p
  - g_N – Landé-g-factor of the core
- Spectroscopic notation: explicit writing of f

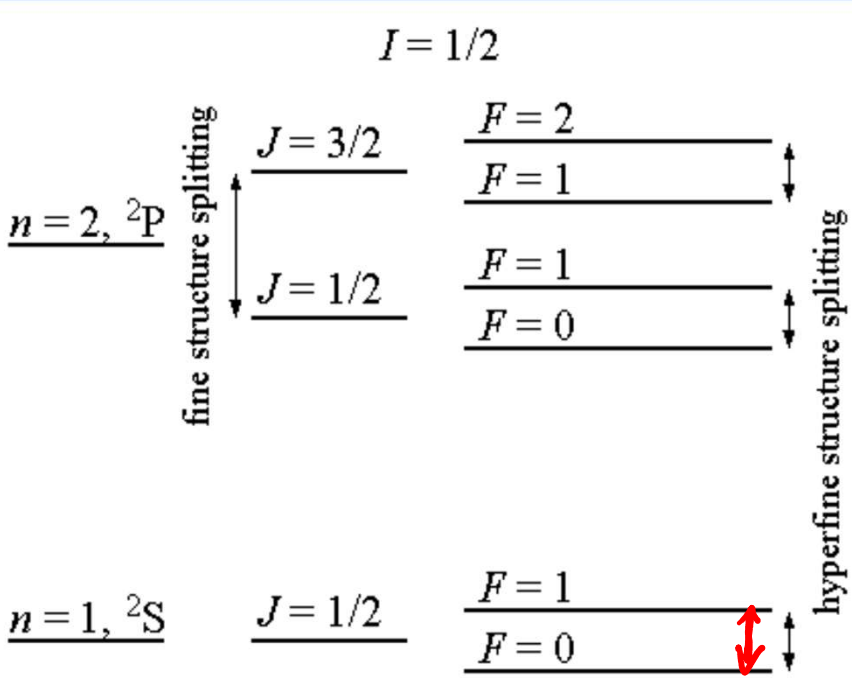
Hyperfine structure splitting

Hyperfine structure

- H-atom
- for $n = 1$ (ground state $l = 0$): ${}^2s_{1/2}: F = 1 \rightarrow 0$
- can be observed for "cold hydrogen"



$F = 1 - 0$



famous 21cm line = 1.42GHz
allows for mapping of the Milky Way

magnetic dipole transition → forbidden

$A = 2.9 \times 10^{-15} s^{-1}$ $(F = 1 \rightarrow F = 0)$

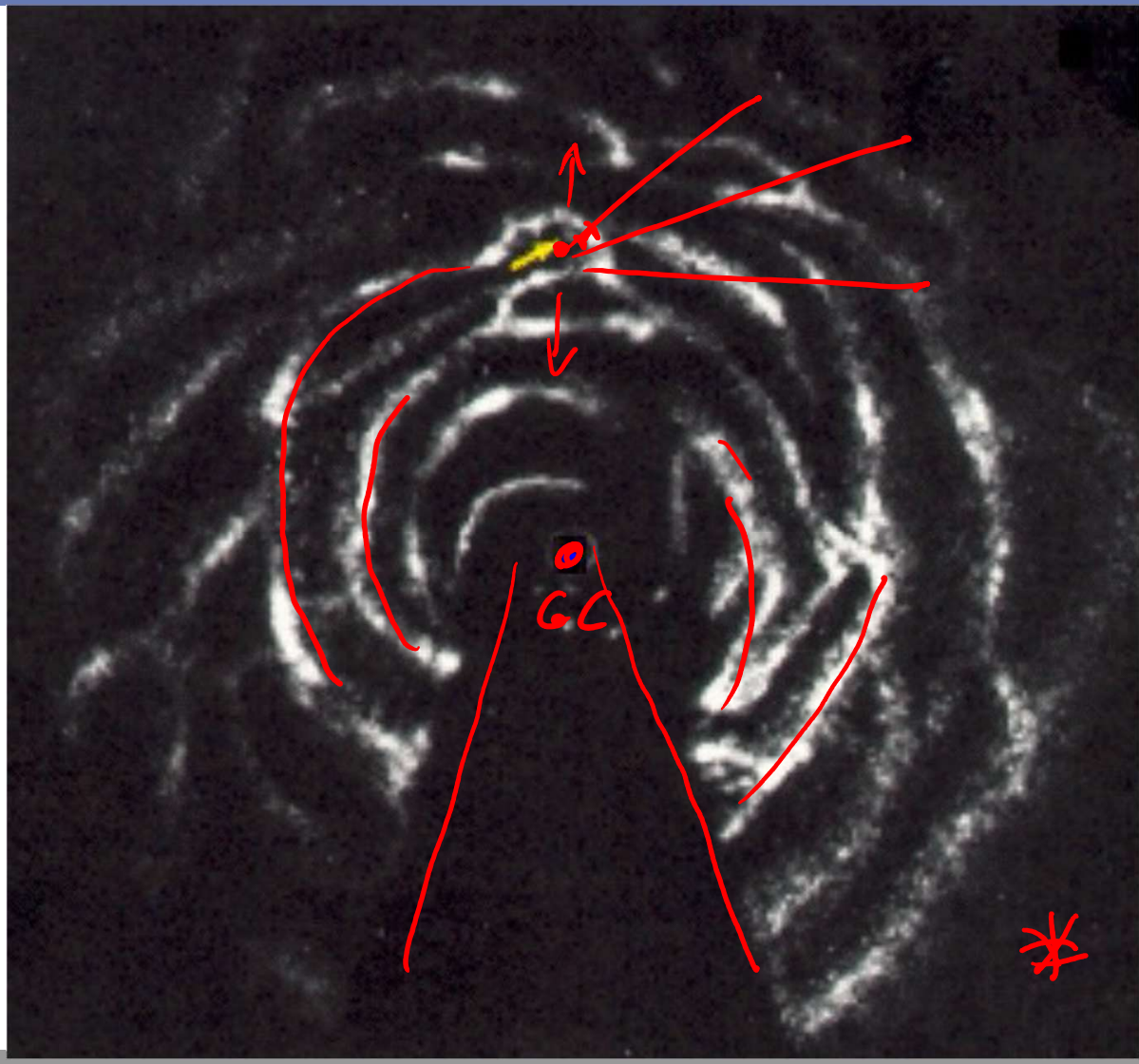
$t = 10^7 a$

Hyperfine structure splitting

Hyperfine structure

- H-atom
- for $n = 1$ (ground state $l = 0$):
  ${}^2S_{1/2}: F = 1 - 0$
- can be observed for “cold hydrogen”
- allows for mapping of
  neutral interstellar gas

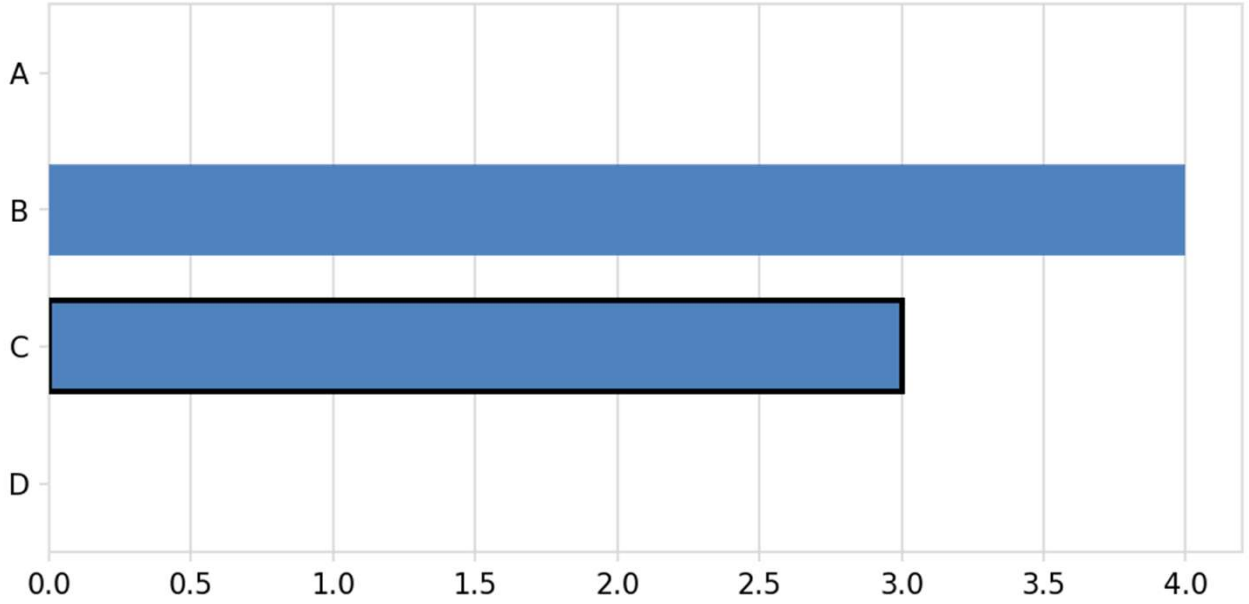
1958 Oort, Kerr, Westerhout :
The galactic system as a spiral nebula



Kurze Frage…

For an allowed electric-dipole transition, the spontaneous emission rate scales with the transition frequency as:

A) $A \propto \omega$
B) $A \propto \omega^2$
C) $A \propto \omega^3$
D) $A \propto \omega^5$



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7 Teilnehmer / Umfrage geschlossen

Kurze Frage…

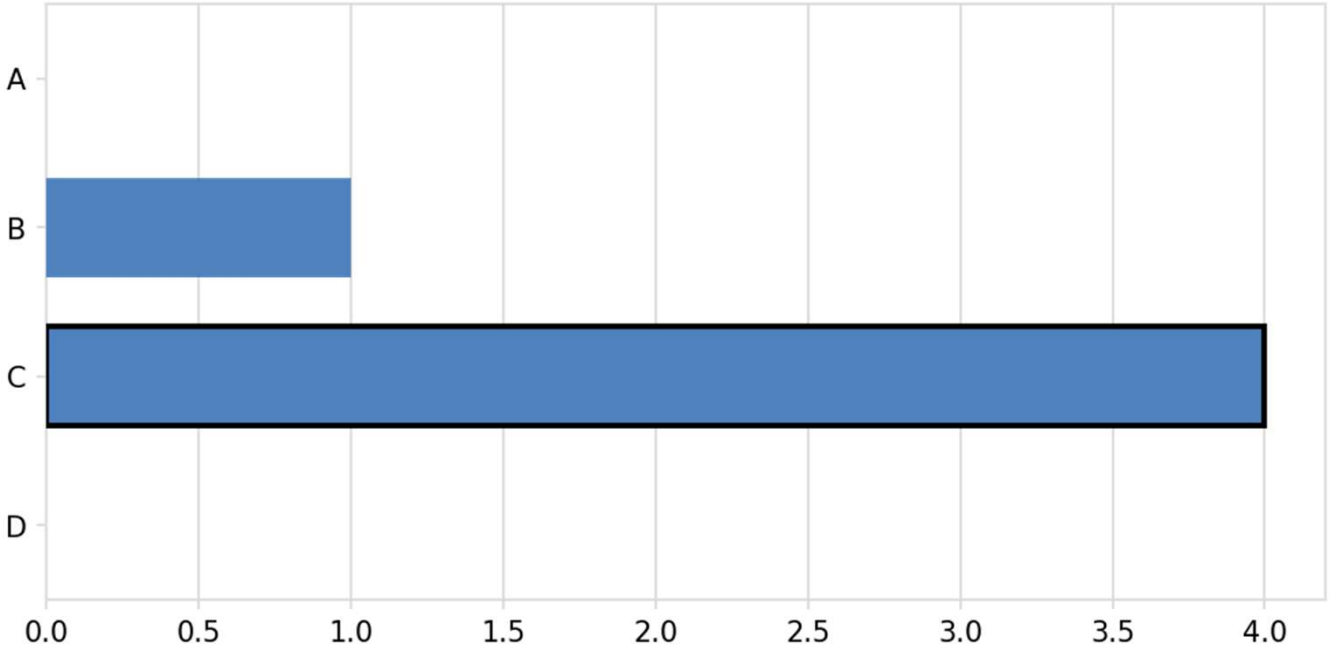
Ignoring electron spin, what is the statistical weight g of the $n = 4$ level of atomic hydrogen?

A) 4
B) 8
C) 16
D) 32

$g_n = n^2$

$n=4 : \ell = 0, 1, 2, 3$

$2\ell + 1$



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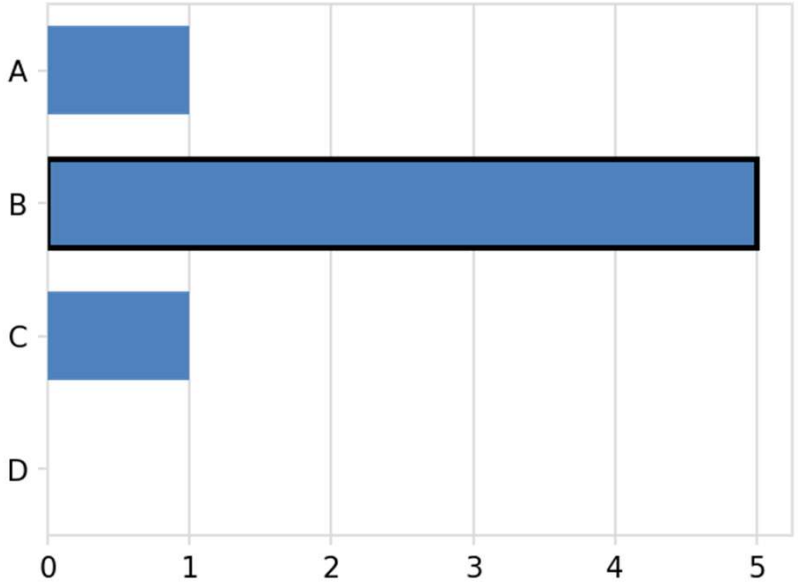
5 Teilnehmer / Umfrage geschlossen

Kurze Frage…

The 21 cm hyperfine line of atomic hydrogen has an extraordinarily long radiative lifetime ($\sim 10^7$ yr). The main reason is:

A) The energy gap is too small compared to typical thermal energies.
B) It is a magnetic-dipole transition — both levels are ${}^2S_{1/2}$, so it is electric-dipole forbidden ($\Delta l = 0$, same parity).
C) The upper state $F = 1$ is not appreciably populated in cold neutral gas.
D) Hyperfine transitions violate conservation of total angular momentum.



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7 Teilnehmer / Umfrage geschlossen

Multiple electrons

- **Arrangement in subshells with increasing energy level**
  = increasing main and orbital quantum numbers n, ℓ
  degeneracy through m_e → number of electrons that can populate subshell

- Example: $C^+ = 1s^2 2s^2 2p^1$
  - 3 subshells occupied
  - Closed shells: Sum of orbital momenta and spins always = 0
    - No contribution to radiative interaction
    - Can be ignored
  - 3rd subshell is open
    - can take 6 electrons: $m_l = -1, 0, 1$ $m_s = -1/2, 1/2$
    - dominates radiative interaction as only electrons in open shell need to be considered

Multiple electrons

- **Coupling between different electrons**
  - Provides additional term to Hamiltonian
  - No analytic solution
  - Approximation through Russell-Saunders coupling
    - Inter-electron coupling stronger than spin-orbit coupling
    - Individual orbital momentums and spins add up $\sum \vec{l} = \vec{L}$ & $\sum \vec{s} = \vec{S}$, $\vec{J} = \vec{L} + \vec{S}$
    - Remember: approximation only → *L,S* are no "good" quantum numbers
    - Different approximation for heavy elements $(\vec{j} - \vec{j}$ coupling$)$ Fe, Co, Ni
- Spectroscopic notation:
  - capital letters for sum over multiple electrons in open shell
  - explicit writing of *F* for sum of total angular momentum including nuclear spin

Multiple electrons

Astrophysically relevant subterms

- Remember H-atom: $^{2s+1}l_j^{(p)}$
- Equivalently for multi-electron atoms
  - Capital letter for sum over multiple electrons in open shell
  - $2S+1$ = "multiplicity" = statistical weight of the state without fine structure

Ground configuration	Terms (in order of increasing energy)	Examples
$\ldots ns^1$	$^2S_{1/2}$	H I, He II, C IV, N V, O VI
$\ldots ns^2$	$^1S_0$	He I, C III, N IV, O V
$\ldots np^1$	$^2P^o_{1/2,3/2}$	C II, N III, O IV
$\ldots np^2$	$^3P_{0,1,2}$, $^1D_2$, $^1S_0$	C I, N II, O III, Ne V, S III
$\ldots np^3$	$^4S^o_{3/2}$, $^2D^o_{3/2,5/2}$, $^2P^o_{1/2,3/2}$	N I, O II, Ne IV, S II, Ar IV
$\ldots np^4$	$^3P_{2,1,0}$, $^1D_2$, $^1S_0$	O I, Ne III, Mg V, Ar III
$\ldots np^5$	$^2P^o_{3/2,1/2}$	Ne II, Na III, Mg IV, Ar IV
$\ldots np^6$	$^1S_0$	Ne I, Na II, Mg III, Ar III

I : neutral
II : singly ionized
III : doubly ionized

CII : C^+

Multiple electrons

Russell-Saunders coupling

wichtigste Kühllinie des warmen neutralen Mediums

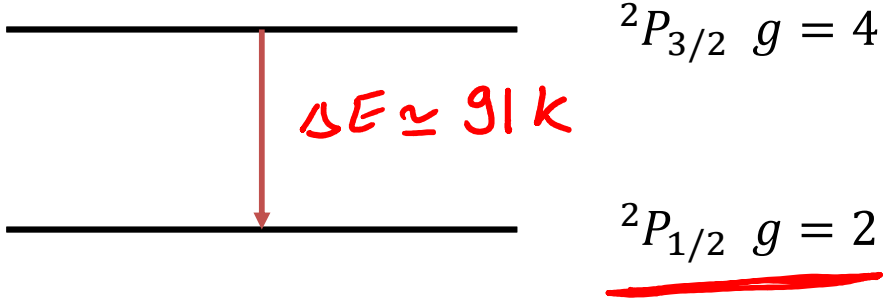
- Solution
- "Hund rules":
  1) higher S → lower energies
  2) higher L → lower energies
  3) lower J and shell ≤ half-full → lower energies
     shell > half-full → higher energies

Geben Sie hier eine Formel ein.

- Example: $C^+ = 1s^2 2s^2 2p^1$
  - 2p electron has $S = 1/2\,, L = 1$
  - gives $J = 1/2,\ (m_J = -1/2\,,\ 1/2)$
    or $J = 3/2,\ (m_J = -3/2, -1/2, 1/2\,,\ 3/2)$ = 2-level system
  - Only 3rd Hund rule applies

$\Delta E = 7.86\text{meV}$
$= 1900.536\text{GHz}$
$= 157.7\mu m$

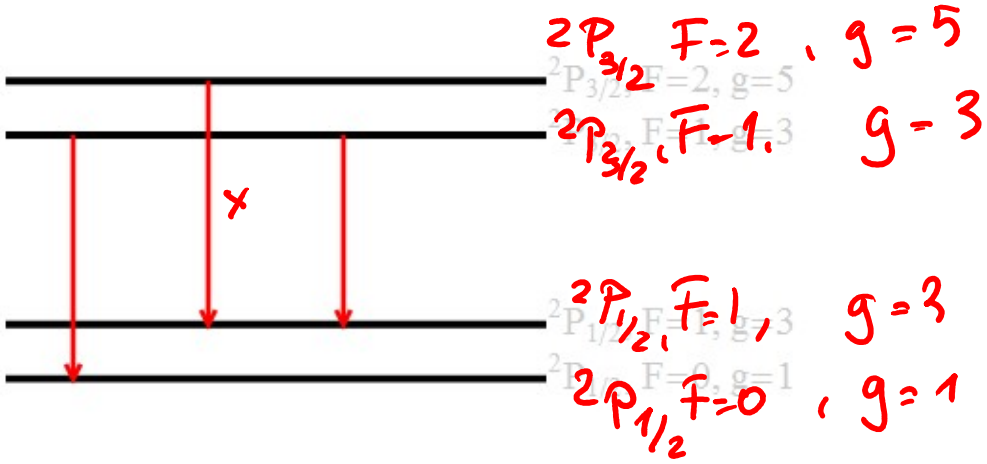
- Equivalent for N^{++}: $\Delta E = 57\mu m$



Hyperfine structure

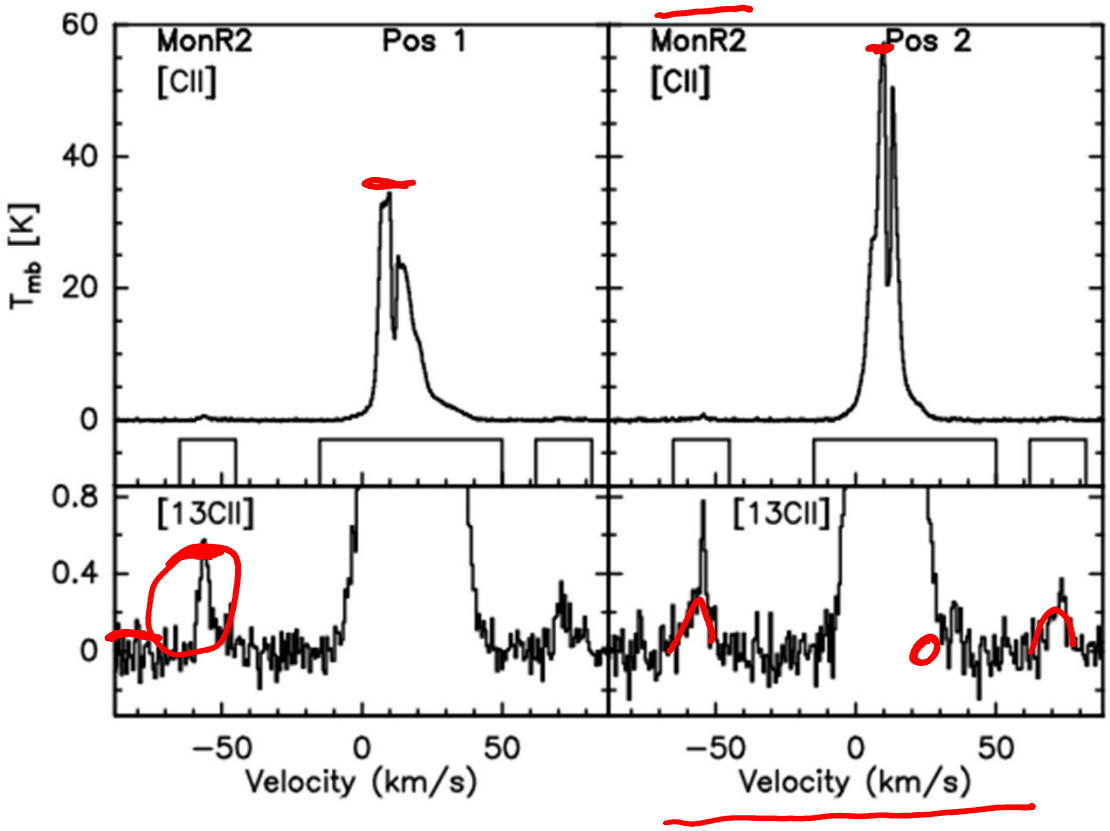
- Superimposed if nuclear spin does not sum up to 0
- Example: $^{13}C^{+}$
  - $I = 1/2$ combines with $J = 1/2$ to $F = 0\ (m_F = 0)\ \ or\ \ F = 1\ \ (m_F = -1,0,1)$
    with $J = 3/2$ to $F = 1\ (m_F = -1,0,1)\ \ or\ \ F = 2(m_F = -2,-1,0,1,2)$
  - C+ levels split up

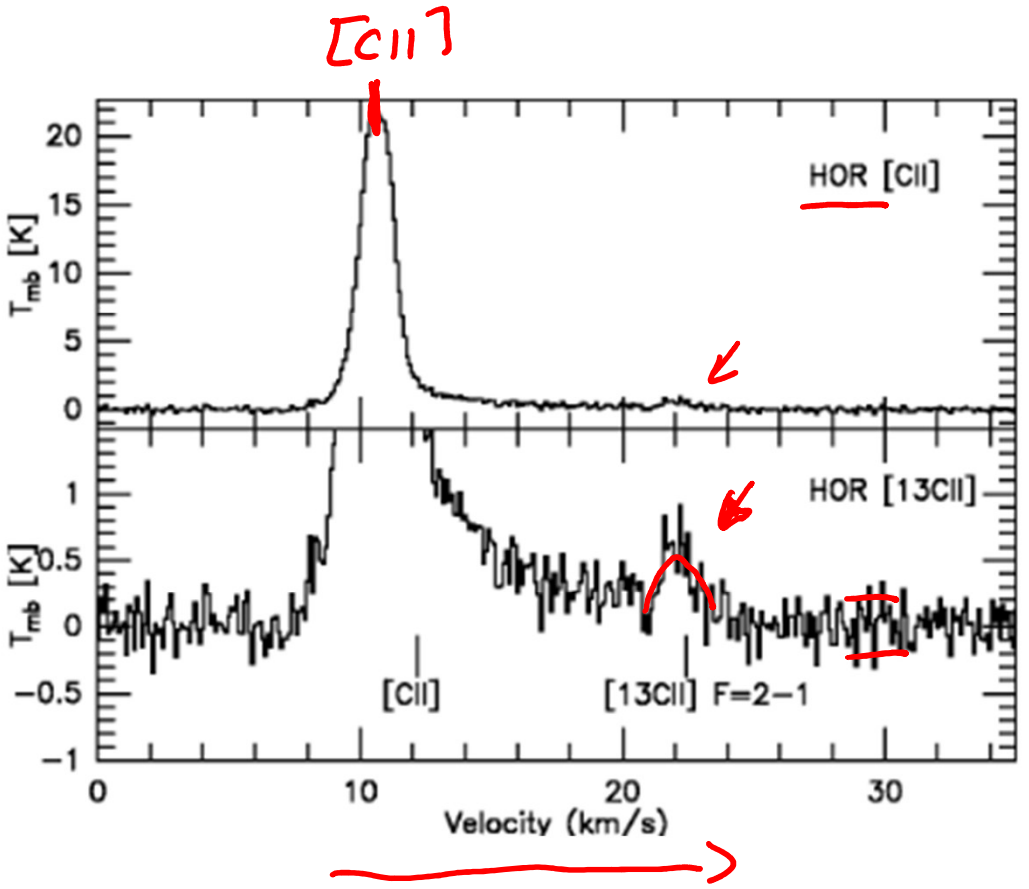
Line	Statistical g_u	Weight g_l	Frequency ν (GHz)	Vel. offset $\delta v_{F\to F'}$ (km s^{-1})	Relative intensity $s_{F\to F'}$
[^{12}C II] $^2P_{3/2}$–$^2P_{1/2}$	4	2	1900.5369	0	–
[^{13}C II] $F = 2 \to 1$	5	3	1900.4661	+11.2	0.625
[^{13}C II] $F = 1 \to 0$	3	1	1900.9500	−65.2	0.250
[^{13}C II] $F = 1 \to 1$	3	3	1900.1360	+63.2	0.125



- 3 possible transitions, F=2→0 is forbidden $(F = 0 \nleftrightarrow F = 0)$

[13CII] Observations





Guevara et al. 2020

Multiple electrons in one subshell

Combination of states

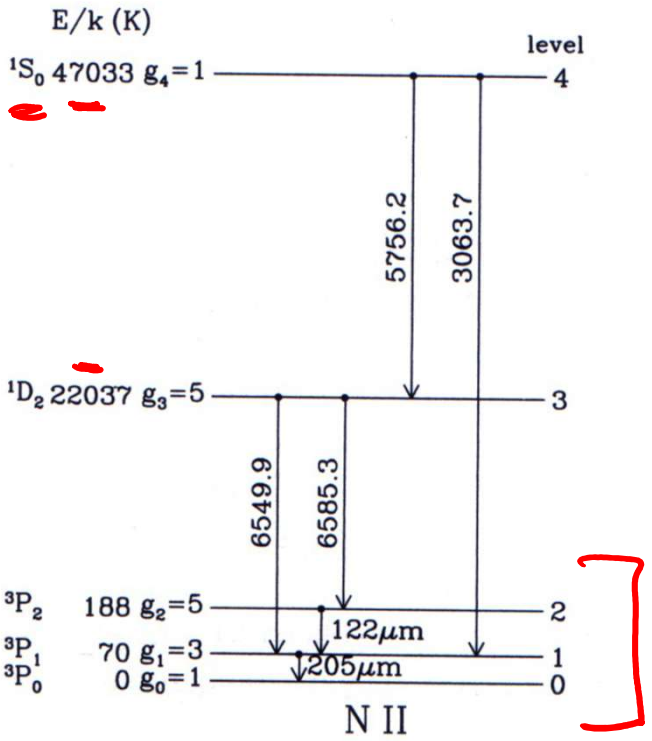
- Example: $N^+ = 1s^2 2s^2 2p^2$
  - Combination of two electrons with *s=1/2* *(m_s=-1/2,1/2), l=1 (m_l=-1,0,1)*
    → gives 6x6=36 possible combinations
    - Pauli exclusion principle:
      - 2 electrons never in same state
      - Electrons indistinguishable, i.e. wave functions antisymmetric with respect of exchange of 2 electrons
  - 15 allowed combinations remaining
    - $l_1 \uparrow\uparrow l_2,\ s_1 \uparrow\downarrow s_2 : L = 2, S = 0 \quad \rightarrow \quad J = 2\ (m_J = -2,-1,0,1,2) \quad \rightarrow {}^1D_2 \quad g = 5$
    - $l_1 \uparrow\downarrow l_2,\ s_1 \uparrow\downarrow s_2 : L = 0, S = 0 \quad \rightarrow \quad J = 0\ (m_J = 0) \quad \rightarrow {}^1S_0 \quad g = 1$
    - $l_1 \perp l_2,\ s_1 \uparrow\uparrow s_2 : L = 1, S = 1 \quad \rightarrow \quad J = 0\ (m_J = 0) \quad \rightarrow {}^3P_0 \quad g = 1$
      $\rightarrow \quad J = 1\ (m_J = -1,0,1) \quad \rightarrow {}^3P_1 \quad g = 3$
      $\rightarrow \quad J = 2\ (m_J = -2,-1,0,1,2) \quad \rightarrow {}^3P_2 \quad g = 5$

Combination of states

$N^+ = 1s^2\,2s^2\,2p^2$

N^+ : 14.3 eV
IP

- "Hund rules" 1) higher S → lower energies
  2) higher L → lower energies
  3) lower J and shell ≤ half-full → lower energies
  shell > half-full → higher energies



The S=0 states need very high excitation energy

→ usually not relevant for ISM physics

→ **treat N^+ as 3-level system**

Only 2 radiative transitions (J=2 → 0 forbidden)

122 μm = 2457 GHz

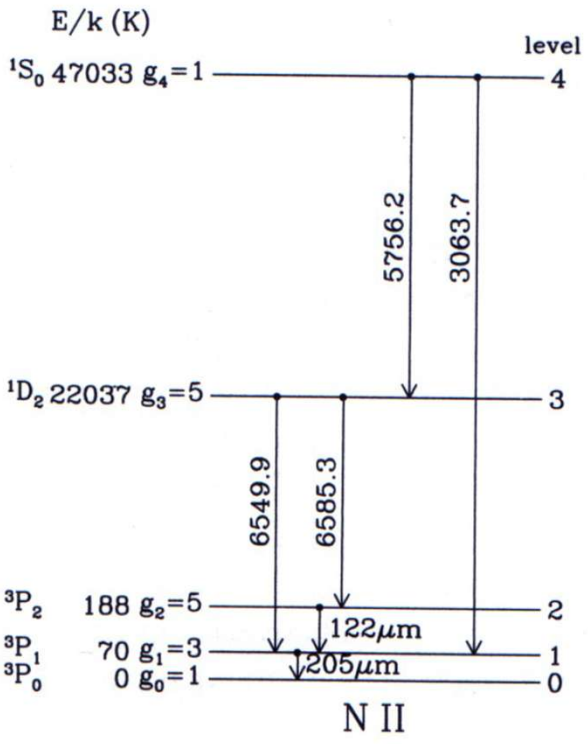
205 μm = 1458 GHz

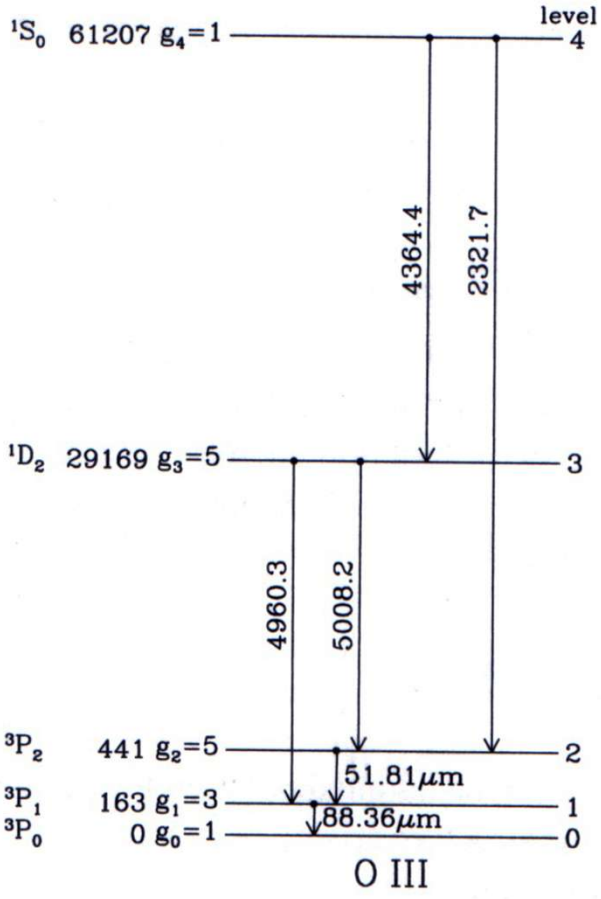
Combination of states

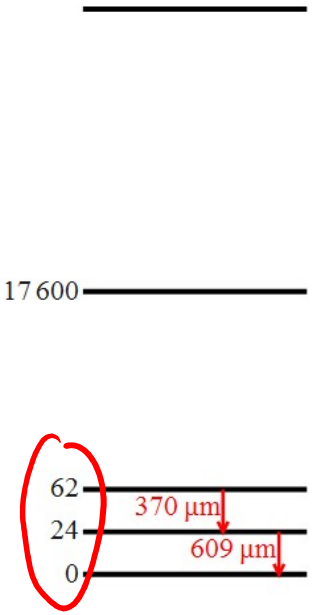
Other atoms with same configuration: $^3P_{0,1,2}$

- $N^+ = O^{++} = C$

traces cold molecular clouds







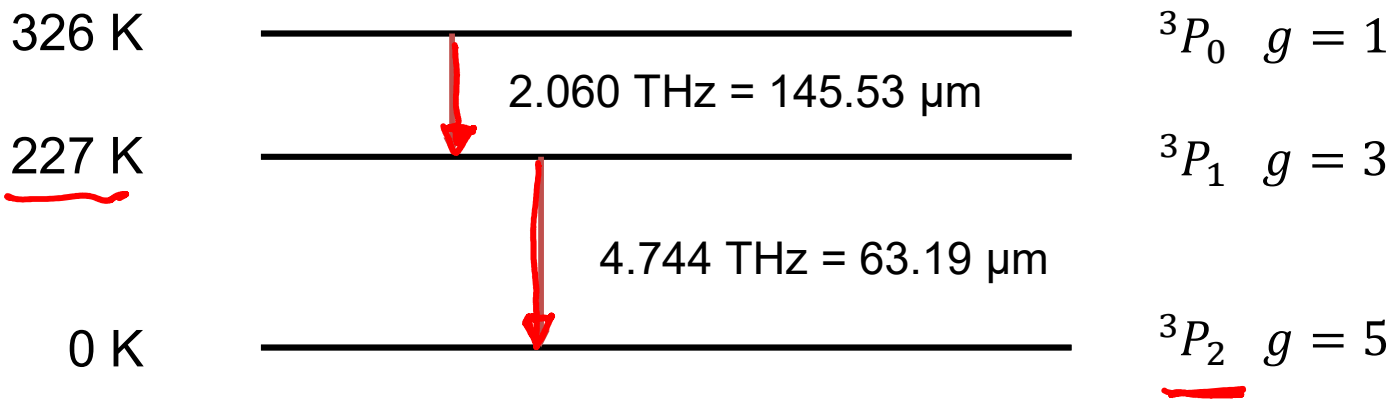
810 GHz
490 GHz

Combination of states

PDR
Photo-Dissociation Region

O = $1s^2 2s^2 2p^4$

- Similar addition of configurations for all 4 electrons
- But: third Hund rule with “shell > half-full” applies → reverse order: $^3P_{2,1,0}$
- Only *S=1* states excited at moderate temperatures (below 20000K)
  - Effectively 3-level system as well
  - Radiative transitions only J = 2 → 1 and J = 1 → 0



$n_{crit} \approx 10^5\ cm^{-3}$

Multiple electrons in one subshell

- **Combination of states**
- O^+ = $1s^2 2s^2 2p^3$
  - 1. Hund rule: higher S → lower energies
  - Ground state → combination of three electrons with same $m_s \rightarrow m_l = -1,0,1$ → single state $S = 3/2,\ L = 0 \rightarrow J = 3/2 \rightarrow {}^4S_{3/2}$, $g = 2J + 1 = 4$
  - Other combinations:

    $S = 1/2,\ L = 2 \rightarrow J = 3/2, 5/2 \rightarrow {}^2D_{3/2,5/2}$

    $S = 1/2,\ L = 1 \rightarrow J = 1/2, 3/2 \rightarrow {}^2P_{1/2,3/2}$
  - 2. Hund rule: higher L → lower energies
    - $^2P_{1/2,3/2}$ above $^2D_{3/2,5/2}$
  - But $^2D_{3/2}$ already 20000K above the $^4S_{3/2}$ state
    - No split of the $^4S_{3/2}$ state from the 3rd Hund rule
    - Minimum fine-structure transition energy 20000K → **"Dark" in the ISM**

Atomic spectroscopy: Summary

Few cases to distinguish:

CNM: 21cm
PDR/cmm: [CII], [OI]
HII: [NII], [OIII], [OII]
warm

- 2-level systems: HI, C^+, N^{++}

- 3-level systems: C, N^+, O^{++}, O
  - $^3P_{0,1,2}$ (C, N^+, O^{++}): ground-state transition has lower frequency
  - $^3P_{2,1,0}$ (O): ground-state transition has higher frequency

- 4-level systems: $^{13}C^+$

- $2p^3$ systems: N, O^+ → “Darkness”
  - no fine-structure transitions excited at temperatures << 20000K